HARD INCLUSIVE PRODUCTION OF A PAIR OF RAPIDITY-SEPARATED HADRONS IN PROTON COLLISIONS

Dmitry Yu. Ivanov[1] and Alessandro Papa[2]

[1] *Sobolev Institute of Mathematics and Novosibirsk State University, 630090 Novosibirsk, Russia*

[2] *Dipartimento di Fisica, Università della Calabria, and Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza, I-87036 Arcavacata di Rende, Cosenza, Italy*

E-mail: d-ivanov@math.nsc.ru, papa@cs.infn.it

We discuss the process $p + p \to h_1 + h_2 + X$, where the identified hadrons h_1 and h_2 have large transverse momenta and are produced in high-energy proton-proton collisions with a large rapidity gap between them. In this case the (calculable) hard part of the reaction receives large higher order corrections $\sim \alpha_s^n \ln^n \Delta y$, which can be accounted for in the BFKL approach. Specifically, we describe in the next-to-leading order the calculation of the vertex (impact-factor) for the inclusive production of the identified hadron.

1 Introduction

The process under consideration is

$$\text{proton}(p_1) \,+\, \text{proton}(p_2) \,\to\, \text{hadron}_1(k_1) \,+\, \text{hadron}_2(k_2) \,+\, X \;.$$

Introducing the Sudakov decomposition for the momentum of each identified hadron,

$$k_h = \alpha_h p_1 + \frac{\vec{k}_h^2}{\alpha_h s} p_2 + k_{h\perp} \;, \qquad k_{h\perp}^2 = -\vec{k}_h^2 \;, \qquad s = 2 p_1 \cdot p_2 \;,$$

we assume that hadrons' transverse momenta are large, $\vec{k}_1^{\,2} \sim \vec{k}_2^{\,2} \gg \Lambda_{\rm QCD}^2$, so that perturbative QCD is applicable. Moreover, we consider the high-energy limit $s = 2p_1 \cdot p_2 \gg \vec{k}_{1,2}^{\,2}$, which opens the way to the BFKL [1] resummation.

Figure 1: Diagrammatic representation of the forward quark (left) and gluon (right) impact factor.

Let us briefly remind the basics of the BFKL approach. In the Regge limit ($s \to \infty$, t not growing with s), the total cross section $A + B \to X$ can be written as (see, for instance, [2])

$$\sigma_{AB} = \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2) \, .$$

This factorization is valid both in the leading logarithmic approximation (LLA), which means resummation of all terms $(\alpha_s \ln s)^n$, and in the next-to-LLA (NLA), which means resummation of all terms $\alpha_s(\alpha_s \ln s)^n$. The Green's function G_ω is process-independent and is determined through the BFKL equation in $D = 4 + 2\epsilon$ dimensions,

$$\omega \, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q} \, K(\vec{q}_1, \vec{q}) \, G_\omega(\vec{q}, \vec{q}_1) \; ,$$

whose kernel is known in the NLA both for forward scattering (i.e. for $t = 0$ and color singlet in the t-channel) [3] and for any fixed (not growing with energy) momentum transfer t and any possible two-gluon color state in the t-channel [4]. As for the process-dependent impact factors (IFs) $\Phi_{A,B}$, only very few have been calculated in the NLA.

The starting point for the calculation in the NLA of the IF relevant for the process under consideration is provided by the IFs for colliding partons [5] (see Fig. 1). We observe that for the LLA IF, there can be only a one-particle intermediate state, whereas for the NLA IF, we can have virtual corrections to the one-particle intermediate state, but also real particle production, with a two-particle intermediate state.

Here are the steps of the calculation:

i) "open" one of the integrations over the phase space of the intermediate state to allow one parton to fragment into a given hadron (see Fig. 2);

ii) use QCD collinear factorization,

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes D_a^h \; + \; f_g \otimes (\text{gluon vertex}) \otimes D_g^h \; ,$$

Figure 2: Diagrammatic representation of the vertex for the identified hadron production for the case of incoming quark (left) or gluon (right).

Figure 3: Diagrammatic representation of the LLA vertex for the case of incoming quark (left) and gluon (right).

iii) project onto the eigenfunctions of the LLA BFKL kernel ((ν, n)-representation),

$$\Phi(\nu, n) = \int d^2\vec{q}\, \frac{\Phi(\vec{q})}{\vec{q}^{\,2}} \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{\gamma - \frac{n}{2}} \left(\vec{q}\cdot\vec{l}\,\right)^n \;, \qquad \gamma = i\nu - \frac{1}{2} \;, \qquad \vec{l}^{\,2} = 0 \;,$$

which is convenient for the numerical convolution with BFKL Green's function.

2 The impact factor in the LLA

The starting point is given by the "inclusive" LLA parton IFs:

$$\Phi_q = g^2 \frac{\sqrt{N^2-1}}{2N} \;, \qquad \Phi_g = \frac{C_A}{C_F}\Phi_q \;, \qquad C_A = N \;, \quad C_F = \frac{N^2-1}{2N} \;.$$

Here the step i) means simply to introduce a delta function (see Fig. 3). Then, QCD collinear factorization leads to

$$\frac{d\Phi^h}{\vec{q}^{\,2}} = \Phi_q \, d\alpha_h \frac{d^{2+2\epsilon}\vec{k}}{\vec{k}^{\,2}} \int_{\alpha_h}^{1} \frac{dx}{x}\, \delta^{(2+2\epsilon)}(\vec{k}-\vec{q}) \left(\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_h}{x}\right) + \sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_h}{x}\right) \right) \;.$$

3 The impact factor in the NLA

Collinear singularities in NLA are to be removed by PDFs' and FFs' renormalization:

$$f_q(x) = f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon}+\ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[P_{qq}(z)f_q(\tfrac{x}{z},\mu_F)+P_{qg}(z)f_g(\tfrac{x}{z},\mu_F)\right]$$

$$f_g(x) = f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon}+\ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[P_{gq}(z)f_q(\tfrac{x}{z},\mu_F)+P_{gg}(z)f_g(\tfrac{x}{z},\mu_F)\right] ,$$

$$D_q^h(x) = D_q^h(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon}+\ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[D_q^h(\tfrac{x}{z},\mu_F)P_{qq}(z)+D_g^h(\tfrac{x}{z},\mu_F)P_{gq}(z)\right]$$

$$D_g^h(x) = D_g^h(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon}+\ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[D_q^h(\tfrac{x}{z},\mu_F)P_{qg}(z)+D_g^h(\tfrac{x}{z},\mu_F)P_{gg}(z))\right] ,$$

where P_{ij} are the Altarelli-Parisi splitting functions and $\frac{1}{\hat\epsilon}=\frac{1}{\epsilon}+\gamma_E-\ln(4\pi)\approx\frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^\epsilon}$. This leads to the following collinear counterterms:

$$\frac{\pi\sqrt{2}\,\vec k^{\,2}}{\Phi_q}\frac{d\Phi^h(\nu,n)|_{\text{coll. c.t.}}}{d\alpha_h d^{2+2\epsilon}\vec k} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon}+\ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_{\alpha_h}^1\frac{dx}{x}\int\limits_{\frac{\alpha_h}{x}}^1\frac{dz}{z}\left(\vec k^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec k\cdot\vec l\right)^n$$

$$\times\left[(1+z^{-2\gamma})P_{qq}(z)\sum_{a=q,\bar q}f_a(x)D_a^h\left(\frac{\alpha_h}{xz}\right)+\left(\frac{C_A}{C_F}+z^{-2\gamma}\right)P_{gq}(z)\sum_{a=q,\bar q}f_a(x)D_g^h\left(\frac{\alpha_h}{xz}\right)\right.$$

$$\left.+(1+z^{-2\gamma})\frac{C_A}{C_F}P_{gg}(z)f_g(x)D_g^h\left(\frac{\alpha_h}{xz}\right)+\frac{C_A}{C_F}\left(\frac{C_F}{C_A}+z^{-2\gamma}\right)P_{qg}(z)f_g(x)\sum_{a=q,\bar q}D_a^h\left(\frac{\alpha_h}{xz}\right)\right].$$

The other counterterm comes from the QCD coupling renormalization and reads

$$\frac{\pi\sqrt{2}\,\vec k^{\,2}}{\Phi_q}\frac{d\Phi(\nu,n)|_{\text{charge c.t.}}}{d\alpha d^{2+2\epsilon}\vec k} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon}+\ln\frac{\mu_R^2}{\mu^2}\right)\left(\frac{11C_A}{6}-\frac{n_f}{3}\right)$$

$$\times\int\limits_{\alpha_h}^1\frac{dx}{x}\left(\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_h}{x}\right)+\sum_{a=q,\bar q}f_a(x)D_a^h\left(\frac{\alpha_h}{x}\right)\right)\left(\vec k^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec k\cdot\vec l\right)^n .$$

In the following we will use the following abbreviation $\frac{\pi\sqrt{2}\,\vec k^{\,2}}{\Phi_q}\frac{d\Phi^h(\nu,n)}{d\alpha_h d^{2+2\epsilon}\vec k}\equiv I$.

Figure 4: Diagrammatic representation of the NLA vertex for the case of incoming quark: real corrections from quark-gluon intermediate state, cases of gluon fragmentation (left) and quark fragmentation (right).

3.1 Quark-initiated subprocess

We have first of all the virtual corrections to the one-particle intermediate state:

$$I_q^V = -\frac{\alpha_s}{2\pi}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{1}{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int\limits_{\alpha_h}^{1}\frac{dx}{x}\sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_h}{x}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n$$

$$\times\left\{C_F\left(\frac{2}{\epsilon}-3\right)-\frac{n_f}{3}+C_A\left(\ln\frac{s_0}{\vec{k}^2}+\frac{11}{6}\right)\right\}+\text{finite terms}\,.$$

Then, we have to consider the "real" corrections from the quark-gluon intermediate state. The starting point is the "inclusive" quark IF,

$$\Phi^{\{QG\}} = \Phi_q g^2 \vec{q}^{\,2}\frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}}\frac{d\beta_1}{\beta_1}\frac{[1+\beta_2^2+\epsilon\beta_1^2]}{\vec{k}_1^{\,2}\vec{k}_2^2(\vec{k}_2\beta_1-\vec{k}_1\beta_2)^2}\left\{C_F\beta_1^2\vec{k}_2^2+C_A\beta_2\left(\vec{k}_1^{\,2}-\beta_1\vec{k}_1\cdot\vec{q}\right)\right\}\,,$$

where $\beta_{1,2}$ and $\vec{k}_{1,2}$ are the relative longitudinal and transverse momenta of the gluon (quark) and $\beta_1+\beta_2=1$, $\vec{k}_1+\vec{k}_2=\vec{q}$.

For gluon fragmentation (see Fig. 4 left), the "parent" parton variables are $\vec{k}=\vec{k}_1$, $\zeta=\beta_1$ and the contribution reads

$$I_{q,g}^R = \tfrac{\alpha_s}{2\pi}\tfrac{1}{\epsilon}\tfrac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\int\limits_{\alpha_h}^{1}\tfrac{dx}{x}\int\limits_{\frac{\alpha_h}{x}}^{1}\tfrac{d\zeta}{\zeta}\textstyle\sum_{a=q,\bar{q}} f_a(x) D_g^h\left(\tfrac{\alpha_h}{x\zeta}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n$$

$$\times P_{gq}(\zeta)\left[\tfrac{C_A}{C_F}+\zeta^{-2\gamma}\right]+\text{finite terms}$$

For quark fragmentation (see Fig. 4 right), the "parent" parton variables are $\vec{k}=\vec{k}_2$,

$\zeta = \beta_2$. The contribution proportional to C_F reads

$$\left(I^R_{q,q}\right)^{C_F} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int\limits_{\alpha_h}^1\frac{dx}{x}\int\limits_{\frac{\alpha_h}{x}}^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar q} f_a(x) D^h_a\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec k^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec k\cdot\vec l\,\right)^n$$
$$\times\left\{C_F\left(\tfrac{2}{\epsilon}-3\right)\delta(1-\zeta)+P_{qq}(\zeta)\left(1+\zeta^{-2\gamma}\right)+\text{finite terms}\right\}\;,$$

while the contribution proportional to C_A reads

$$\left(I^R_{q,q}\right)^{C_A} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\int\limits_{\alpha_h}^1\frac{dx}{x}\int\limits_{\frac{\alpha_h}{x}}^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar q} f_a(x) D^h_a\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec k^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec k\cdot\vec l\,\right)^n$$
$$\times C_A\,\delta(1-\zeta)\ln\frac{s_0}{\vec k^2}+\text{finite terms}\,.$$

3.2 Gluon-initiated subprocess

The virtual corrections to the one-particle intermediate state are

$$I^V_g = -\frac{\alpha_s}{2\pi}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{1}{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int\limits_{\alpha_h}^1\frac{dx}{x}f_g(x)D^h_g\left(\frac{\alpha_h}{x}\right)\left(\vec k^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec k\cdot\vec l\,\right)^n\frac{C_A}{C_F}$$

$$\times\left\{C_A\left(\ln\frac{s_0}{\vec k^2}+\frac{2}{\epsilon}-\frac{11}{6}\right)+\frac{n_f}{3}\right\}+\text{finite terms}\,.$$

For the "real" corrections from quark-antiquark intermediate state, the starting point is the corresponding contribution to the "inclusive" gluon IF ($T_R = 1/2$),

$$\Phi^{\{Q\bar Q\}} = \Phi_g g^2\vec q^{\,2}\frac{d^{2+2\epsilon}\vec k_1}{(2\pi)^{3+2\epsilon}}d\beta_1 T_R\left(1-\frac{2\beta_1\beta_2}{1+\epsilon}\right)\left\{\frac{C_F}{C_A}\frac{1}{\vec k_1^{\,2}\vec k_2^{\,2}}+\beta_1\beta_2\frac{\vec k_1\cdot\vec k_2}{\vec k_1^{\,2}\vec k_2^{\,2}(\vec k_2\beta_1-\vec k_1\beta_2)^2}\right\},$$

where $\beta_{1,2}$ and $\vec k_{1,2}$ are the relative longitudinal and transverse momenta of the quark (antiquark) and $\beta_1+\beta_2=1$, $\vec k_1+\vec k_2=\vec q$.

For quark (or antiquark) fragmentation (see Fig. 5 left) the "parent" parton variables are $\vec k=\vec k_1$, $\zeta=\beta_1$ and the contribution reads

$$I^R_{g,q} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\int\limits_{\alpha_h}^1\frac{dx}{x}\int\limits_{\frac{\alpha_h}{x}}^1\frac{d\zeta}{\zeta}f_g(x)\sum_{a=q,\bar q} D^h_a\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec k^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec k\cdot\vec l\,\right)^n\frac{C_A}{C_F}$$
$$\times P_{qg}(\zeta)\left[\frac{C_F}{C_A}+\zeta^{-2\gamma}\right]+\text{finite terms}\,.$$

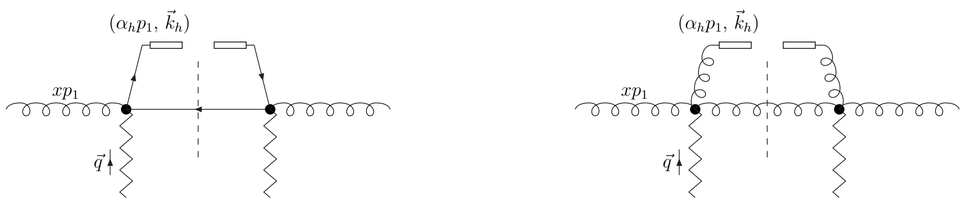

Figure 5: Diagrammatic representation of the NLA vertex for the case of incoming gluon: real corrections from quark-antiquark intermediate state, case of quark fragmentation (left) and from two-gluon intermediate state, case of gluon fragmentation (right).

For "real" corrections from the two-gluon intermediate state, the starting point is the corresponding contribution to the "inclusive" gluon IF,

$$\Phi^{\{GG\}} = \Phi_g g^2 \vec{q}^{\,2} \frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}} d\beta_1 \frac{C_A}{2}\left[\frac{1}{\beta_1}+\frac{1}{\beta_2}-2+\beta_1\beta_2\right]$$
$$\times\left\{\frac{1}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}}+\frac{\beta_1^2}{\vec{k}_1^{\,2}(\vec{k}_2\beta_1-\vec{k}_1\beta_2)^2}+\frac{\beta_2^2}{\vec{k}_2^{\,2}(\vec{k}_2\beta_1-\vec{k}_1\beta_2)^2}\right\}\,,$$

where $\beta_{1,2}$ and $\vec{k}_{1,2}$ are the relative longitudinal and transverse momenta of the two gluons and $\beta_1+\beta_2=1$, $\vec{k}_1+\vec{k}_2=\vec{q}$. We can have only gluon fragmentation to be counted with a factor of 2 (see Fig. 5 right). The result is

$$I^R_{g,g} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int_{\alpha_h}^1\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^1\frac{d\zeta}{\zeta}f_g(x)D_g^h\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n\frac{C_A}{C_F}$$
$$\times\left\{P_{gg}(\zeta)\left(1+\zeta^{-2\gamma}\right)+\delta(1-\zeta)\left[C_A\left(\ln\frac{s_0}{\vec{k}^2}+\frac{2}{\epsilon}-\frac{11}{3}\right)+\frac{2n_f}{3}\right]\right\}+\text{finite terms}\,.$$

4 Final result and discussion

One can verify that all UV and IF divergences cancel, leading to

$$\vec{k}_h^2\,\frac{d\Phi^h(\nu,n)}{d\alpha_h d^2\vec{k}_h} = 2\,\alpha_s(\mu_R)\sqrt{\frac{C_F}{C_A}}\left(\vec{k}_h^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}_h\cdot\vec{l}\right)^n\left\{\int_{\alpha_h}^1\frac{dx}{x}\left(\frac{x}{\alpha_h}\right)^{2\gamma}\left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_h}{x}\right)\right.\right.$$
$$\left.+\sum_{a=q,\bar{q}}f_a(x)D_a^h\left(\frac{\alpha_h}{x}\right)\right]+\frac{\alpha_s\left(\mu_R\right)}{2\pi}\int_{\alpha_h}^1\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^1\frac{d\zeta}{\zeta}\left(\frac{x\,\zeta}{\alpha_h}\right)^{2\gamma}$$

$$\times \left[\frac{C_A}{C_F} f_g(x) D_g^h \left(\frac{\alpha_h}{x\zeta} \right) C_{gg}\left(x,\zeta\right) + \sum_{a=q,\bar{q}} f_a(x) D_a^h \left(\frac{\alpha_h}{x\zeta} \right) C_{qq}\left(x,\zeta\right) \right.$$
$$\left. + \sum_{a=q,\bar{q}} f_a(x) D_g^h \left(\frac{\alpha_h}{x\zeta} \right) C_{qg}\left(x,\zeta\right) + \frac{C_A}{C_F} f_g(x) \sum_{a=q,\bar{q}} D_a^h \left(\frac{\alpha_h}{x\zeta} \right) C_{gq}\left(x,\zeta\right) \right] \Bigg\} .$$

The explicit form of the coefficient functions will be given elsewhere [7].

To summarize, we have discussed the NLA calculation of the IF the forward production of an identified hadron from an incoming quark or gluon, emitted by a proton. This is a necessary ingredient for the hard inclusive production of a pair of rapidity-separated identified hadrons at LHC. We have given our result in the (ν, n)-representation, which is the most convenient for the numerical determination of the cross section [6]. We have shown that soft and virtual infrared divergences cancel each other, whereas the IR collinear ones are compensated by the PDFs' and FFs' renormalization counterterms, the remaining UV divergences being taken care of by the QCD coupling renormalization.

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